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Analysis of heat transfer of power-law fluid along a vertical stretching cylinder

Research Article

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Abstract: The aim of this paper is to study the heat transfer enhancement of power-law fluid over a vertical stretching cylinder. The model is governed by partial differential equations (PDEs) which is then converted into a coupled system of non-linear ordinary differential equations (ODEs). Here both velocity and temperature profiles have been constructed for different physical parameters such as, Reynold number, Prandtl number, Eckert number, unsteadiness, mixed convective and magnetic-parameters. The obtained results have been compared with the existing literature that reveals the accuracy of the proposed scheme. Moreover, it has been noticed during the study that the power-law energy is getting enhanced corresponding to these parameters, which can be viewed in graphical simulations. In addition, the numerical computations of local-skin friction coefficient and Nusselt number are tabulated.

Keywords: Heat transfer • Power law • Magnetic parameter • BVP4C

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1. Introduction

A wide number of mixed fluids are used in modern technologies and some fluids act substantially differently than Newtonian fluids. Among these fluids are mixtures such as suspensions, paints, emulsions, and greece. In daily life items, such as pharmaceutics, personal care products, honey, blood, toothpaste, paint, biological fluids *etc* do not obey Newton's viscosity law. These fluids are known as non-Newtonian fluids. Ketchup is the most obvious example, as it becomes runnier when shaken. Non-Newtonian fluids have a great impact on our industry. As a result, several experiments have been carried out in an effort to identify the physical properties of non-Newtonian

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fluids. Because of the wide range of non-Newtonian fluids, several fluid models have been constructed to investigate their properties. Non-Newtonian fluids cannot be underestimated, but Newtonian fluids must never be neglected.

Because of their diverse applications, the study of flow and heat transfer of non-Newtonian fluids across a stretching surface has garnered a lot of interest in recent years in engineering problems, such as chemical engineering and particularly, manufacturing of plastic and rubber sheets, hot rolling, continuous cooling, and fibres spinning, exotic lubricants as well as suspension solutions. Certain of these fluids feature power-law behaviour, in which the viscosity is a function of shear rate, and have been dubbed Ostwald-de Waele fluids by some writers. Jain and Bohra [1] introduced the entropy generation on MHD slip flow over a stretching cylinder with heat generation/absorption. Shampine *et al.* [2] investigated Solving boundary value problems for ordinary differential equations in MATLAB with bvp4c. Mushtaq *et al.* [3] formulated mixed convection flow of second grade fluid along a vertical stretching flat surface with variable surface temperature. They observed that both the skin friction coefficient and heat transfer coefficient drop near the leading edge as the elasticity parameter increases, but this behavior reverses in the downstream regime. Prasad *et al.* [4] investigated the influence of thermal buoyancy on a non-Newtonian power law fluid flow across a continuous vertical stretching sheet. They considered the velocity at the surface and temperature distribution at surface to be linear in their variation. They concluded that for the constant value of the buoyancy parameter, the local skin friction coefficient and local Nusselt number drops as the magnetic parameter increases. Patil *et al.* [5,6] studied mixed convection flow over a vertical power-law stretching sheet and unsteady effects on mixed convection boundary layer flow from a permeable slender cylinder due to non-linearly power law stretching, respectively. Megahed [7] examined the heat transfer properties of a non-Newtonian power-law fluid induced by a non-linearly impermeable stretching vertical sheet in the presence of thermal radiation and a constant heat flux. He observed that raising the value of power-law index, mixed convection, and radiation parameter results in a decrease in the skin friction coefficient while local Nusselt number increases with the increasing value of Prandtl number or mixed convective parameter. . Naseer *et al.* [8] presented numerical study of convective heat transfer on the power-law fluid over a vertical exponentially stretching cylinder. Ferdows *et al.* [9] investigated magnetohydrodynamics flow and heat transfer of a power-law non-Newtonian nanofluid ($Cu - H_20$) over a vertical stretching sheet. Ahmed *et al.* [10] proposed axisymmetric flow and heat transfer over an unsteady stretching sheet in power-law fluid. Shojaei *et al.* [11] proposed hydrothermal analysis of Non-Newtonian second grade fluid flow on radiative stretching cylinder with Soret and Dufour effects. Halifi *et al.* [12] reviewed numerical solution of bio-magnetic power-law fluid flow and heat transfer in a Channel. Mahmoud *et al.* [13] investigated Slip velocity effect on a non-Newtonian power-law fluid over a moving permeable surface with heat generation. Bibi *et al.* [14] elucidated the impacts of different shapes of nanoparticles on SiO_2 nanofluid flow and heat transfer in a liquid film over a stretching sheet. Sahu *et al.* [15] considered the momentum and heat transfer in a power-law fluid from a continuous moving plate with non-uniform surface velocity distributions. Elahi *et al.*[16] examined numerical Simulation of Heat Transfer

Development of Nanofluids in a Thin Film over a Stretching Surface. Kudenatti [17] described hydrodynamic flow of non-Newtonian power-law fluid past a moving wedge or a stretching sheet: a unified computational approach. In the light of the context, the manipulation of flow and heat transfer rate of power-law fluid over an unsteady vertically stretching cylinder along with convective boundary conditions has been proposed which has not been published or studied anywhere earlier.

Nomenclature

u, w	Velocity components in r and z directions (ms^{-1})
U_w	Fluid velocity (ms^{-1})
T_w	Surface temperature (K)
T_∞	Ambient fluid temperature (K)
T_0	Slit temperature (K)
κ_f	Thermal conductivity of base fluid $(WK^{-1}m^{-1})$
c_p	Specific heat of fluid $(Jkg^{-1}K^{-1})$
γ	Biot number
Nu	Nusselt number
Re	Reynold number
Pr	Prandtl number
M	Magnetic parameter
K_0	Consistency thermal coefficient
λ	Mixed convection parameter
B_0	Uniform magnetic field
A	Unsteadiness parameter
α_f	Thermal diffusion of base fluid (m^2s^{-1})
ρ_f	Density of base fluid (kgm^{-3})
μ_f	Dynamic viscosity of base fluid $(kgms^{-1})$
ν_f	Kinematic viscosity of base fluid (m^2s^{-1})
α	Thermal conductivity of fluid
C	Curvature

2. Mathematical Formulation

Consider two dimensional flow of unsteady and an incompressible non-Newtonian power-law fluid over a vertical stretching cylinder. Assume that a fluid is moving with uniform velocity $U_w = \frac{cz}{1-\alpha t}$ over a vertical stretching cylinder in z direction based on applied transverse magnetic field, given by $B(t) = \frac{B_0}{1-\alpha t}$ which is assumed to be of variable kind as shown in Figure 1. The surface temperature of the cylinder is T_w with an ambient fluid

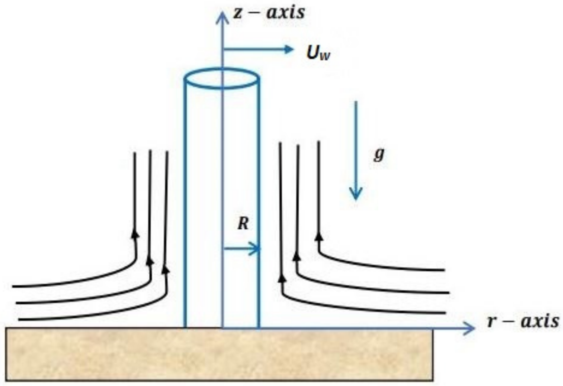

Figure 1. **Physical representation of the problem**

temperature is T_∞ such that $T_w > T_\infty$. In cylindrical coordinate system, u and w are assumed to be the velocity components in radial-and z-directions respectively. Under these assumptions, the governing boundary-layer equations, based on the balance laws of mass, linear momentum and energy equations for investigation are given, as [18]:

$$\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z} = \frac{1}{\rho r}\frac{\partial}{\partial r}\left(rk\left|\frac{\partial w}{\partial r}\right|^{n-1}\frac{\partial w}{\partial r}\right)-\frac{\sigma}{\rho}(B(t))^2 w+g\beta(T-T_\infty), \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z} = \frac{k}{\rho c_p}\left(-\frac{\partial w}{\partial r}\right)^{n+1}+\frac{k_o}{\rho c_p}\left|\frac{\partial T}{\partial r}\right|^{n-1}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right), \tag{3}$$

subject to the boundary conditions [19,20]

$$\begin{aligned} u=0, w=U_w, T=T_w, at r=R, \\ w\to 0, T\to T_\infty, as r\to\infty, \end{aligned} \tag{4}$$

where n is the power-law index, ρ is the fluid density, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, σ is the electric conductivity of the medium, c_p is the specific heat constant, while b and c are the dimensional constant.

Introducing the similarity transformations, as [21]

$$T=T_\infty+\left(\frac{bz}{1-\alpha t}\right)\theta(\eta), \eta=\left(\frac{r^2-R^2}{2zR}\right)Re^{\frac{1}{n+1}}, \psi=zRURe^{-\frac{1}{n+1}}f(\eta), \tag{5}$$

ψ is a stream function that describes the flow pattern, defined as

$u=-\frac{1}{r}\frac{\partial\psi}{\partial z}andw=\frac{1}{r}\frac{\partial\psi}{\partial r}$, satisfy directly the continuity equation (1).

Using (5) in the governing equations (2-3) together with the boundary conditions (4), leads to

$$n(1+2\eta\ C)^{\frac{n+1}{2}}(-f''(\eta))^{n-1}f'''(\eta)-(n+1)C(1+2\eta\ C)^{\frac{n-1}{2}}(-f'')^{n}+\left(\frac{2n}{n+1}\right)f(\eta)f''(\eta)$$
$$-f'^{2}-Mf'(\eta)-A\left[f'(\eta)+\left(\frac{2-n}{n+1}\right)\eta\ f''(\eta)\right]+\lambda\ \theta(\eta)=0, \tag{6}$$
$$(1+2\eta\ C)^{\frac{n+1}{2}}(-\theta'(\eta))^{n-1}\theta''(\eta)-PrA\left[\theta+\left(\frac{2-n}{n+1}\right)\eta\theta'(\eta)\right]-2C(1+2\eta\ C)^{\frac{n-1}{2}}(-\theta')^{n}$$
$$-Pr\left(f'\theta-\frac{2n}{n+1}\theta' f\right)+Ec\ Pr(-f'')^{n+1}(1+2\eta\ C)^{\frac{n+1}{2}}=0, \tag{7}$$

subject to the boundary conditions

$$f(0)=0, f'(0)=1, \theta(0)=1, at\eta=0,$$
$$f'(\eta)\to 0, \theta(\eta)\to 0as\eta\to\infty. \tag{8}$$

The dimensionless form of other physical parameters can be written, more instructively, as
$Ec=\frac{U^2}{c_p(T_w-T_\infty)}$ (Eckert number), $M=\frac{B_0^2\sigma}{\rho c}$ (magnetic parameter), $Pr=kc_pk_0^{-1}b^{1-n}\ C^{n-1}$ (Prandtl number), and $A=\frac{\alpha}{c}$ (unsteadiness parameter).

The local skin friction coefficient and Nusselt number are, much important in engineering and other fields of sciences, defined as

$$C_f=\frac{2\tau_w}{\rho\ U_w^2}, and Nu=\frac{z\ q_w}{k\ (T_w-T_\infty)}, \tag{9}$$

where

$$\tau_w=k\left(\frac{\partial w}{\partial r}\right)^n_{r=R}, and q_w\ =\ -k\left(\frac{\partial T}{\partial r}\right)_{r=R}. \tag{10}$$

Using the transformations (5) in (9-10), yields the following dimensionless form

$$\frac{1}{2}\ C_fRe^{\frac{1}{n+1}}=-(-f''(0))^n,\quad Nu\ Re^{\frac{-1}{n+1}}=-\theta'(0). \tag{11}$$

3. Method of Solution

Different researchers developed different algorithms, such as shooting method, Homotopy analysis method, and finite difference methods to determine the solution of nonlinear system of equations. In this paper, BVP4C scheme is proposed to solve the boundary value problem (6-8). The detailed discussion on error and convergence analysis of the proposed scheme can be found in [22,23]. The faster convergence with less error, accept not only two-point but multi-point BVPs directly with better accuracy, and dealing with singular BVPs *etc* are the features of the proposed scheme that signify the scheme to others available in the literature. In order to find the numerical solution of a nonlinear system (6-8), the following set of first order ordinary differential equations is obtained by assuming

$$y_1 = f, \tag{12}$$
$$y_1' = y_2, \tag{13}$$
$$y_2' = y_3, \tag{14}$$
$$y_3' = g, \tag{15}$$
$$\theta = y_4, \tag{16}$$
$$y_4' = y_5, \tag{17}$$
$$y_5' = h, \tag{18}$$
$$y_2(0) = 1, y_1(0) = 0, y_4(0) = 1, \tag{19}$$
$$y_2(\eta) \to 0, y_4(\eta) \to 0, as\eta \to \infty, \tag{20}$$

where

$$g = \frac{1}{n(1+2\eta\ C)^{\frac{n+1}{2}}(-y_3)^{n-1}}\left[A\left(y_2+\left(\frac{2-n}{n+1}\eta\ y_3\right)\right)+(n+1)C(1+2\eta\ C)^{\frac{n-1}{2}}(-y_3)^n - \frac{2n}{n+1}y_1y_3+M\ y_2+y_2^2-\lambda\ y_4\right],$$

and

$$h = \frac{1}{(1+2\eta\ C)^{\frac{n+1}{2}}(-y_5)^{n-1}}\left[Pr\ A\left(y_4+\left(\frac{2-n}{n+1}\eta\ y_5\right)\right)+Pr\ \left(y_2y_4-\frac{2n}{n+1}y_1y_5\right) - Ec\ Pr(-y_3)^{n+1}(1+2\eta\ C)^{\frac{n+1}{2}}+2C(1+2\eta\ C)^{\frac{n-1}{2}}(-y_5)^n\right]$$

Thus, the above system of ODEs is then solved using BVP4C on MATLAB. The obtained results are compared with existing ones and presented in Table 1 that confirms the reliability of the scheme.

Table 1. Comparison of numerical results when $M=\gamma=0$ and $Pr=0.7$

A	Sharidan *et al.* [24]	Mukhopadhyay *et al.* [25]	Chamkha *et al.* [26]	Present results
0.8	−1.261042	−1.26152	−1.261499	−1.261042
1.2	−1.377722	−1.378052	−1.377850	−1.3777231

4. Results and Discussion

The aim of this portion is to analyze the effects of both velocity and temperature profiles by varying the different physical parameters. These effects have been presented in the form of graphical simulation and numerical computation, as well. Moreover, the numerical values of local-skin friction and-Nusselt number have also been computed and tabulated in Tables 2-3. In addition, the temperature profile $\theta(\eta)$ is demonstrated graphically for pseudoplastic fluid ($n < 1$), Newtonian fluid ($n = 1$), and dilatant fluid ($n > 1$) along with various values of Prandtl number Pr, Eckert number Ec and curvature parameter C.

The Prandtl number Pr of the fluid plays an important role on the temperature profile $\theta(\eta)$. It is observed that, in Figure 2 as the values of Pr is getting increase progressively the temperature of fluid decreases. Lower Prandtl fluids have higher thermal conductivities and (thicker thermal boundary layer structures) allowing heat to permeate from the cylinder faster than higher Pr fluids (thinner boundary layer). As a result, the Prandtl number can be employed to accelerate cooling in conductive fluids. It can also be seen that the temperature profile improves as the surface convection parameter is increased. Figure 3 depicts the effect of the curvature parameter C on temperature distribution. An increase in C decreases the surface contact area of fluid particles, resulting in less resistance and an increase in average velocity. Temperature rises because the Kelvin temperature is defined by an average kinetic energy. Figure 4 demonstrates that the Eckert number Ec raises the temperature of the fluid, and it is also clear that the Eckert number has a significant impact on the temperature profile. Furthermore, greater values of Ec accelerate the rise of fluid temperature since Ec accelerates the fluid particles.

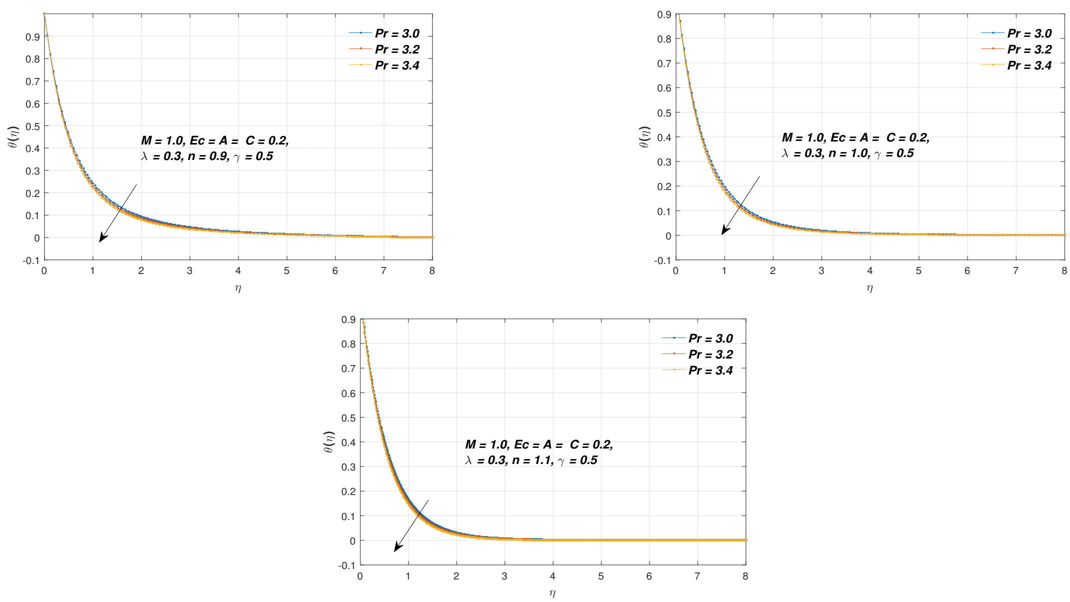

Figure 2. **Temperature distribution for various value of Prandtl number** Pr

Further, the values of local skin friction coefficient C_f are recorded in Tables 2 for different values of the physical parameters. For $n > 0$, raising the Eckert number Ec increased skin friction. It is interesting to note that the skin friction decreases for curvature parameter and prandtl number.

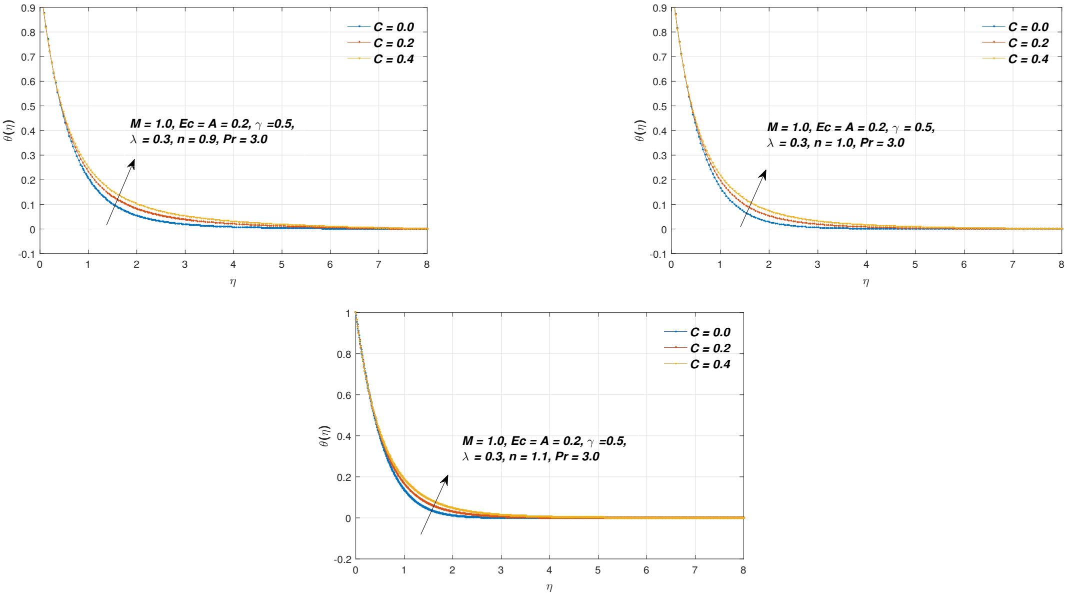

Figure 3. **Temperature distribution for various value of Curvature** C

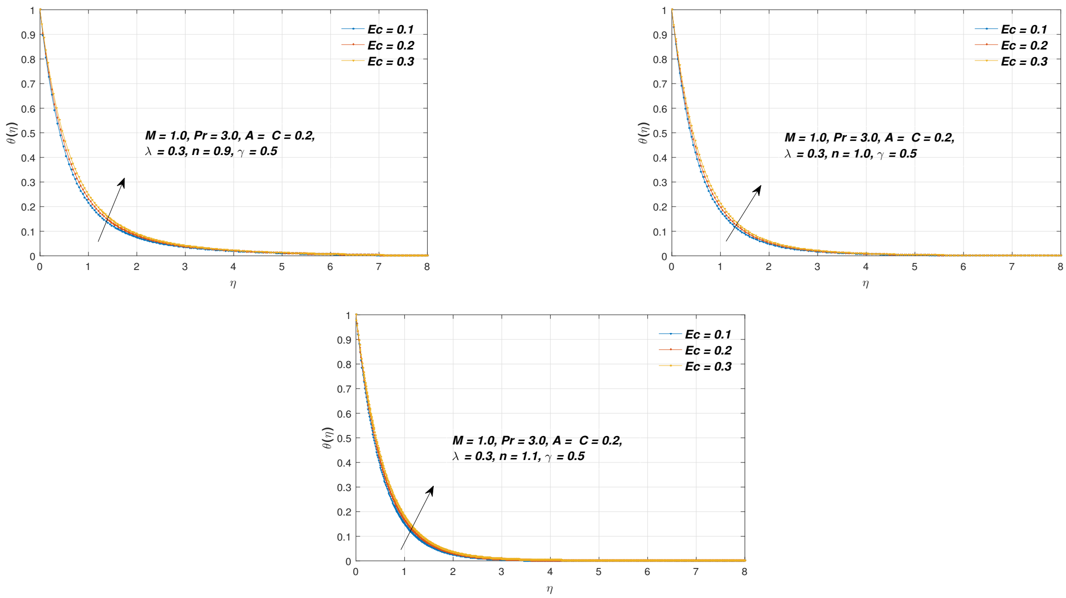

Figure 4. **Temperature distribution for various value of Eckert number** Ec

Finally, Table 3 presented a fully characterize the behaviour of relevant physical quantities such as, the local Nusselt number with changes in the power-law index parameter n, the Eckert number Ec, curvature parameter C, and the Prandtl number Pr, respectively. It can be seen that increasing the index parameter n resulting the increase in Nusselt number. The local Nusselt number decrease as Eckert varies in $0.1 \leq Ec \leq 0.3$. Additionally, an increase in Prandtl number, leads to an increase in the local Nusselt number. This is because a fluid with a

Table 2. **Numerical values of skin-friction coefficient**

Physical parameters			$-\frac{1}{2}\ C_f Re^{\frac{1}{n+1}}$		
C	Pr	Ec	$n = 0.9$	$n = 1.0$	$n = 1.1$
0.0	3.0	0.2	1.3995090	1.4034420	1.4070002
0.2	-	-	1.4854511	1.4901646	1.4943177
0.4	-	-	1.5675509	1.5732526	1.5782448
	3.0	-	1.4854511	1.4901646	1.4943177
-	**3.2**	-	1.4868109	1.4915466	1.4956830
-	**3.4**	-	1.4880742	1.4928243	1.4969402
		0.1	1.4876980	1.4924328	1.4965756
-	-	**0.2**	1.4854511	1.4901646	1.4943177
-	-	**0.3**	1.4832240	1.4879092	1.4919992

higher Prandtl number has a higher heat capacity and hence increases heat transport. Furthermore, the rise in Eckert and curvature parameter yield the decrease in local Nusselt number.

Table 3. **Numerical values of Nusselt number**

Physical parameters			$Nu\ Re^{\frac{-1}{n+1}}$		
C	Pr	Ec	$n = 0.9$	$n = 1.0$	$n = 1.1$
0.0	3.0	0.2	1.6140052	1.6683049	1.7109266
0.2	-	-	1.6568742	1.7147538	1.7612612
0.4	-	-	1.7071610	1.7623152	1.8121100
	3.0		1.6568742	1.7147538	1.7612612
-	**3.2**	-	1.7148645	1.7716797	1.8167214
-	**3.4**	-	1.7710055	1.8266825	1.8702214
		0.1	1.8152508	1.8642603	1.9022101
-	-	**0.2**	1.6568742	1.7147538	1.7612612
-	-	**0.3**	1.5007229	1.5661655	1.6198888

5. Conclusion

This paper deals with the study of flow and heat transfer enhancement in the power law fluid modle subjected to the convective boundary conditions. The numerical simulations of the proposed model are completed using BVP4C package on Matlab and interpreted in the following lines, as

- The growing values of curvature and Eckert number give rise in temperature profile, while the reverse trend is observed against the Prandtl number.
- On varing the Eckert number, the Nusselt number decreases but opposite trend is found in Prandtl number.
- It should be noted that both Nusselt number and skin friction coefficient are getting improved for the power law index n, when $0.9 \leq n \leq 1.1$.

Finally, in view of these lines, the development in temperature distribution of power law fluid model reveals that the proposed scheme "BVP4C" is much reliable and efficient to achive our goal.

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